Name:

College Algebra (Math 1023)
Practice Exam # 1

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This examination contains five problems which are worth 20 points each. The extra credit problem is worth 10 additional points. The use of notebook computers, calculators, cell phones, or any electronic devices during the test is not permitted.

Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	ExCred	Total Score

Problem 1. (True/False)
Circle the letter corresponding to the best answer.

Let f be a function, and suppose that $f(a) = b_1$ and $f(a) = b_2$. Then $b_1 = b_2$.

(T) True **(F)** False

Let f be a function, and suppose that $f(a_1) = f(a_2)$. Then $a_1 = a_2$.

(T) True **(F)** False

There exists a function f such that $f(2) = 3$ and $f(3) = 17$.

(T) True **(F)** False

There exists a function f such that $f(5) = 3$ and $f(5) = 2$.

(T) True **(F)** False

There exists a constant function f such that $f(5) = 3$ and $f(6) = 2$.

(T) True **(F)** False

There exists a linear function such that $f(5) = 3$ and $f(7) = 2$.

(T) True **(F)** False

There exists a linear function such that $f(5) = 3$, $f(6) = 4$, and $f(7) = 7$.

(T) True **(F)** False

There exists a quadratic function such that $f(5) = 3$, $f(6) = 4$,and $f(7) = 7$.

(T) True **(F)** False

There exists a linear function whose range is the set of positive real numbers.

(T) True **(F)** False

There exists a quadratic function whose range is the set of positive real numbers.

(T) True **(F)** False

Problem 2. (Computation) Compute the following functions. All correct answers are polynomials, which should be expressed in standard form (combine like terms and write in decreasing power order).

(a) Let $f(x) = x^3 + 2x + 1$ and $g(x) = x^4 + 2x^3 - 3x^2 - 2x + 3$. Find the sum $f(x) + g(x)$.

(b) Let $f(x) = x^2 - 3$ and $g(x) = x^2 + 2x + 3$. Find the product $f(x)g(x)$.

(c) Let $f(x) = x^2 - 14x + 49$. and $g(x) = \sqrt{x} + 5$. Find the composition $g(f(x))$.

(d) Let $f(x) = x^3$. Find $f(f(f(x)))$.

(e) Let $f(x) = mx + k$ and $g(x) = ax^2 + bx + c$. Find $f(g(x)) - g(f(x))$.

Problem 3. (Solving Equations) Find all solutions to the following equations (best five of seven).

(a) $x^2 + 5 = 0$

(b) $3x - 10 = 7x + 3$

(c) $\frac{x}{x+1} = x - 1$

(d) $\sqrt{x^2 + 49} = x + 7$

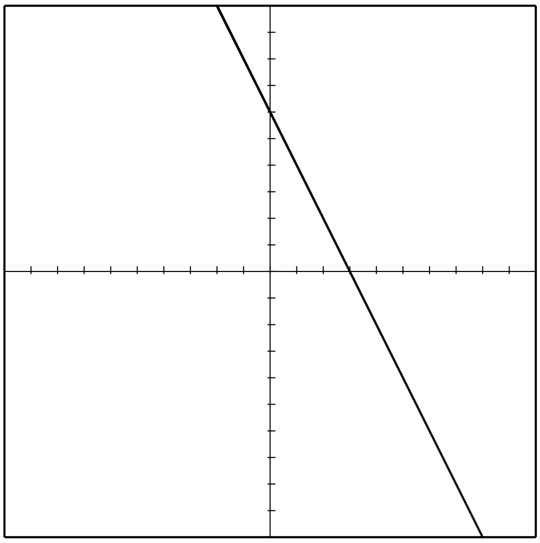
(e) $\sqrt{x^2 - 49} = x - 7$

(f) $x^2 - 7x + 1 = 0$

(g) $x^3 = 9x$

Problem 4. (Linear Functions)

(a) Consider the following graph of a linear function. Fill in all of the information to the right of the graph. To do this, identify the y-intercept and the x-intercept of the line, and use this to compute the slope.



Standard Form:

m: **b:**

Slope:

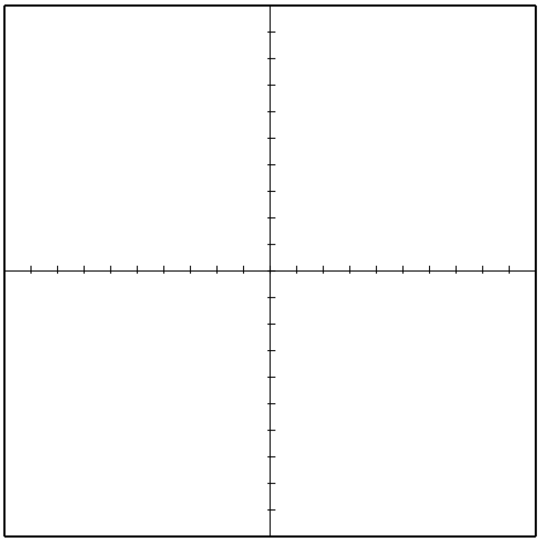
y**-intercept:**

x**-intercept:**

(b) Consider the equation

$$\boxed{6x = 4y + 8.}$$

Solve for y to make a linear function whose graph is a line. Find the standard form $y = mx + b$ of the linear function, and identify the numbers m and b. Find the slope, the y-intercept, and the x-intercept. Sketch the graph of the function.



Standard Form:

m: **b:**

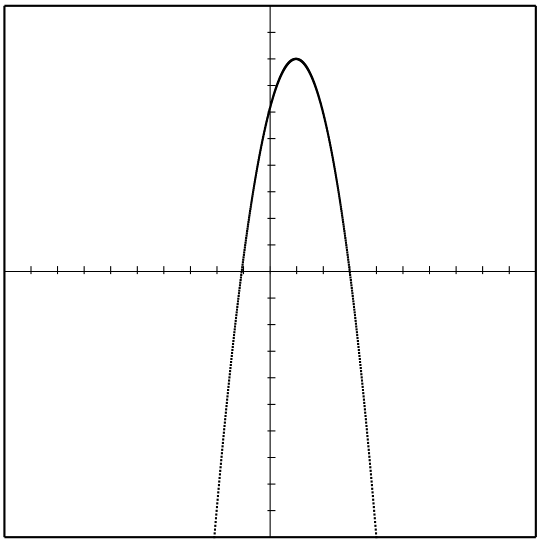
Slope:

y**-intercept:**

x**-intercept:**

Problem 5. (Quadratic Functions)

(a) Consider the following graph of a quadratic function. Fill in all of the information to the right of the graph. To do this, identify the vertex (h, k), the y-intercept $(0, c)$, and the x-intercepts of the function. Plug $x = 0$ and $y = c$ into $y = a(x - h)^2 + k$ and solve for a. Now you have the values of a, h, k, and the shifted form $y = a(x - h)^2 + k$. Multiply this out to find the general form $y = ax^2 + bx + c$.



General Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Zeros:

y-intercept:

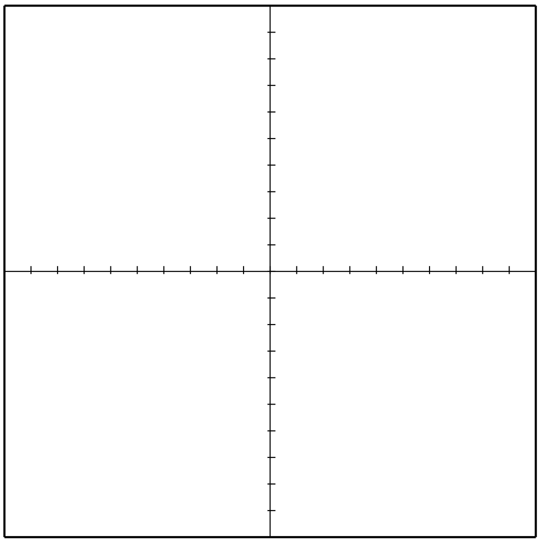
x-intercept(s):

Vertex:

(b) Consider the equation

$$\boxed{y = x^2 - 4x - 5.}$$

This is a quadratic function whose graph is a parabola. Find the general form $y = ax^2 + bx + c$ and the shifted form $y = a(x - h)^2 + k$ of the function. Identify the numbers a, b, c, h, k. Find the zeros of the function. Find the y-intercept, the x-intercepts, and the vertex. Sketch the graph of the function.



General Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Zeros:

y-intercept:

x-intercept(s):

Vertex:

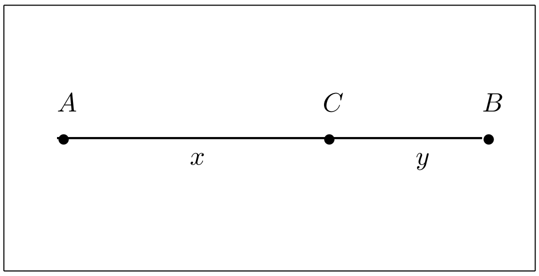
Problem 6. (Extra Credit) A number which fascinated the ancient Greeks was the *golden ratio*, which today is normally denoted by the Greek letter Φ.

If P and Q are points in a plane, let $\overline{PQ}$ denote the line segment from P to Q, and let $|PQ|$ denote the length of this line segment.

Let A and B be points in a plane, and let C be a point on the line segment $\overline{AB}$. We say that C produces is a *golden section* of $\overline{AB}$ if

$$|AB| : |AC| \quad :: \quad |AC| : |BC|,$$

that is, if the ratio of the distance from A to B to the distance from A to C is equal to the ratio of the distance from A to C to the distance from C to B. This common ratio is Φ.



Assume that C produces a golden section of $\overline{AB}$, and that $|AB| = 1$, $|AC| = x$ and $|CB| = y$. Then

$$\boxed{\Phi = \frac{1}{x} = \frac{x}{y}.}$$

Use the picture to find another equation involving x and y. Then find Φ.